Physics 2DL Homework 1 Solutions

3.10 a) Given a measurement error of ± 0.005 in :

Let d be the thickness of 1 card

Measure 1 deck $\Big\}$ $52d = 0.590 \pm 0.005$ in

$$\Rightarrow d = \frac{0.590 \pm 0.005}{52} = \underline{0.0113 \pm 0.0001 \text{ in}}$$

b) Let n be the number of cards required for uncertainty of 2×10^{-5} in

Uncertainty of thickness for n cards : $2\times10^{-5} n \geq 0.005$ in

$\Rightarrow n \geq 250$ cards

$\therefore$ you need $\frac{250}{52} \approx 5$ decks.

3.22 a) $P = IV$ with $I = 2.10 \pm 0.02$ A , $V = 1.02 \pm 0.01$ V

$$\Rightarrow P = 2.14\,W. \qquad \frac{\delta P}{P} = \sqrt{\left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2} = 0.0137$$

$\Rightarrow \delta P = 0.0293\,W$

$\therefore$ $\underline{P = 2.14 \pm 0.03\,W}$

b) $R = V/I = 0.486\,\Omega$ $\qquad \frac{\delta R}{R} = \sqrt{\left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2} = 0.0137$ as for (a)

$\Rightarrow \delta R = 0.0068\,\Omega$

$\therefore$ $\underline{R = 0.486 \pm 0.007\,\Omega}$

3.46 $q = xy + x^2/y$; $x = 6.0 \pm 0.1$, $y = 3.0 \pm 0.1$

$\Rightarrow q = 30$.

$$(\delta q)^2 = \left(\frac{\partial q}{\partial x}\delta x\right)^2 + \left(\frac{\partial q}{\partial y}\delta y\right)^2$$

with $\delta x = 0.1$, $\delta y = 0.1$

$$\frac{\partial q}{\partial x} = y + \frac{2x}{y} = 7 \qquad \frac{\partial q}{\partial y} = x - \frac{x^2}{y^2} = 2$$

$$\therefore (\delta q)^2 = (7 \times 0.1)^2 + (2 \times 0.1)^2 = 0.53$$

$$\Rightarrow \delta q = 0.73$$

$$\therefore q = 30 \pm 0.7$$

4.16 a) $\{g_i\} = \{9.9, 9.6, 9.5, 9.7, 9.8\}$ m/s²

Mean $\bar{g} = \Sigma g_i / N = 9.70$ m/s²

Standard deviation $\sigma_g = \sqrt{\frac{\sum (g_i - \bar{g})^2}{N-1}} = 0.158$ m/s²

$\Rightarrow$ standard error of mean $\sigma_{\bar{g}} = \frac{\sigma_g}{\sqrt{N}} = 0.07$ m/s²

$\therefore g = 9.70 \pm 0.07$ m/s²

b) Compare to $g_{acc} = 9.80$ m/s² :

$$t = \frac{|\bar{g} - g_{acc}|}{\sigma_{\bar{g}}} = \frac{|9.7 - 9.8|}{0.07} \approx 1.4$$

i.e. 1.4 standard deviations from accepted value : satisfactory

4.23 $$e = K\eta^{3/2}$$

$$\delta e = \frac{\partial e}{\partial \eta}\delta\eta \Rightarrow \frac{\delta e}{e} = 3/2\,\frac{\delta\eta}{\eta}$$

$$\therefore \frac{\delta e}{e} = 3/2 \times 0.4\% = 0.6\%$$

4.25 $u = f\lambda$ $\therefore \left(\frac{\delta u}{u}\right)^2 = \left(\frac{\delta f}{f}\right)^2 + \left(\frac{\delta\lambda}{\lambda}\right)^2$

a) $\lambda = 11.2$ cm, $\delta\lambda = 0.5$cm
$f = 3000$ Hz, $\delta f/f = 0.01$

$$\therefore \frac{\delta u}{u} = \sqrt{\left(\frac{0.5}{11.2}\right)^2 + 0.01^2} = 0.046$$

$$\Rightarrow u = 33600 \pm 1545.6 \text{ cm/s}$$
or 336 ± 15 m/s

Since $(\delta\lambda/\lambda) \approx 0.045 \gg (\delta f/f)$, the random error in λ dominates the final uncertainty in u.

b) Now $\delta\lambda = 0.1$cm $\Rightarrow \frac{\delta\lambda}{\lambda} = \frac{0.1}{11.2} = 0.0089$
and $\delta f/f = 0.03$

$$\Rightarrow \frac{\delta u}{u} = \sqrt{(0.0089)^2 + (0.03)^2} \approx 0.03$$

$$\Rightarrow u = 336 \pm 10 \text{m/s}$$

Now the uncertainty in u is dominated by $\delta f/f$, the systematic error in frequency.